



Summarizing Your Data

Key Info

So now you have collected your raw data, and you have results from multiple trials of your experiment. How do you go from piles of raw data to summaries that can help you analyze your data and support your conclusions?

Fortunately, there are mathematical summaries of your data that can convey a lot of information with just a few numbers. These summaries are called **descriptive statistics**. The following discussion is a brief introduction to the two types of descriptive statistics that are generally most useful:

1. Summaries that calculate the "middle" or "average" of your data; these are called **measures of central tendency**, and
2. Summaries that indicate the "spread" of the raw measurements around the average, called **measures of dispersion**.

Measures of Central Tendency: Mean, Median, and Mode

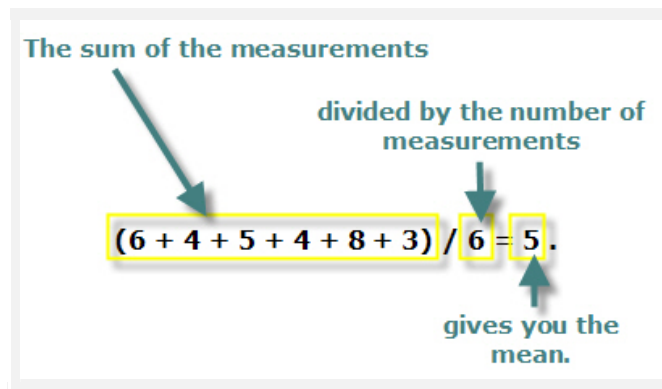
In most cases, the first thing that you will want to know about a group of measurements is the "average." But what, exactly, is the "average?" Is it the mathematical average of our measurements? Is it a kind of half-way point in our data set? Is it the outcome that happened most frequently? Actually, any of these three measures could conceivably be used to convey the central tendency of the data. Most often, the mathematical average or **mean** of the data is used, but two other measures, the **median** and **mode** are also sometimes used.

We'll use a plant growth experiment as an example. Let's say that the experiment was to test whether plants grown in soil with compost added would grow faster than plants grown in the same soil without compost. Let's imagine that we used six separate pots for each condition, with one plant per pot. (In many cases, your project will have more than six trials. We are using fewer trials to keep the illustration simpler.) One of the growth measures chosen was the number of leaves on each plant. Suppose that the following results were obtained:

Plant Growth Without Compost (# of leaves/plant)	Plant Growth With Compost (# of leaves/plant)
6	5
4	9
5	9
4	11
8	8
3	6

Mean

The *mean* value is what we typically call the "average." You calculate the mean by adding up all of the measurements in a group and then dividing by the number of measurements. For the "without compost" case, the mean is 5, as you can see in the illustration below.



For the "with compost" case, the mean is 8. Use the numbers in the table above to do the calculation for yourself to confirm that this is correct.

Median and Mode

The easiest way to find the median and the mode is to first sort each group of measurements in order, from the smallest to the largest. Here are the values sorted in order:

Plant Growth Without Compost (# of leaves/plant)	Plant Growth With Compost (# of leaves/plant)
3	5
4	6
4	8
5	9
6	9
8	11

The median is a value at the midpoint of the group. More explicitly, exactly half of the values in the group are smaller than the median, and the other half of the values in the group are greater than the median. If there are an odd number of measurements, the median is simply equal to the middle value of the group, when the values are arranged in ascending order. If there are an even number of measurements (as here), the median is equal to the mean of the two middle values (again, when the values are arranged in ascending order). For the "without compost" group, the median is equal to the mean of the values of the 3rd and 4th values, which happen to be 4 and 5:

$$\text{median of "without compost" group} = (4 + 5) / 2 = 4.5 .$$

Notice that, by definition, three of the values (3, 4, and 4) are below the median, and the other three values (5, 6, and 8) are above the median. What is the median of the "with compost" group?

The mode is the value that appears most frequently in the group of measurements. For the "without compost" group, the mode is 4, because that value is repeated twice, while all of the other values are only represented once. What is the mode of the "with compost" group?

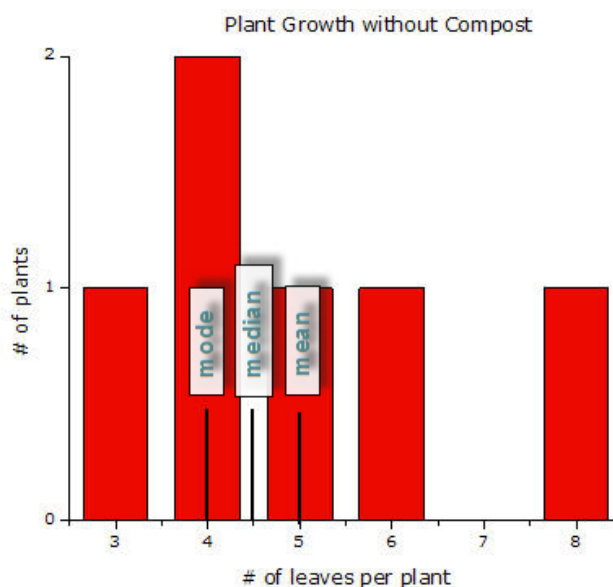
It is entirely possible for a group of data to have no mode at all, or for it to have more than one mode. If all values occur with the same frequency (for example, if all values occur only once), then the group has no mode. If more than one value occurs at the highest frequency, then each of those values is a mode. Here is an example of a group of raw data with two modes:

16, 26, 26, 28, 29, 32, 34, 36, 38, 39, 40, 41, 41, 43, 44, 50 .

The two modes of this data set are 26 and 41, since each of those values appears twice, while all the other values appear only once. A data set with two modes is sometimes called "bimodal." Multi-modal data sets are also possible.

Mean, Median, or Mode: Which Measure Should I Use?

What's the difference between these measures? When would you choose to use one in preference to another? The illustration below shows the mean, median, and mode of the "without compost" data sample on a graph. The x-axis shows the number of leaves per plant. The height of each bar (y-axis) shows the number of plants that had a certain number of leaves. (Compare the graph with the data in the table, and you will see that all of the raw data values are shown in the graph.) This graph shows why the mean, median, and mode are all called measures of central tendency. The data values are spread out across the horizontal axis of the graph, but the mean, median, and mode are all clustered towards the center. Each one is a slightly different measure of what happened "on average" in the experiment. The mode (4) shows which number of leaves per plant occurred most frequently. The median (4.5) shows the value that divides the data points in half; half of the values are lower and half of the values are higher than the median. The mean (5) is the arithmetic average of all the data points.

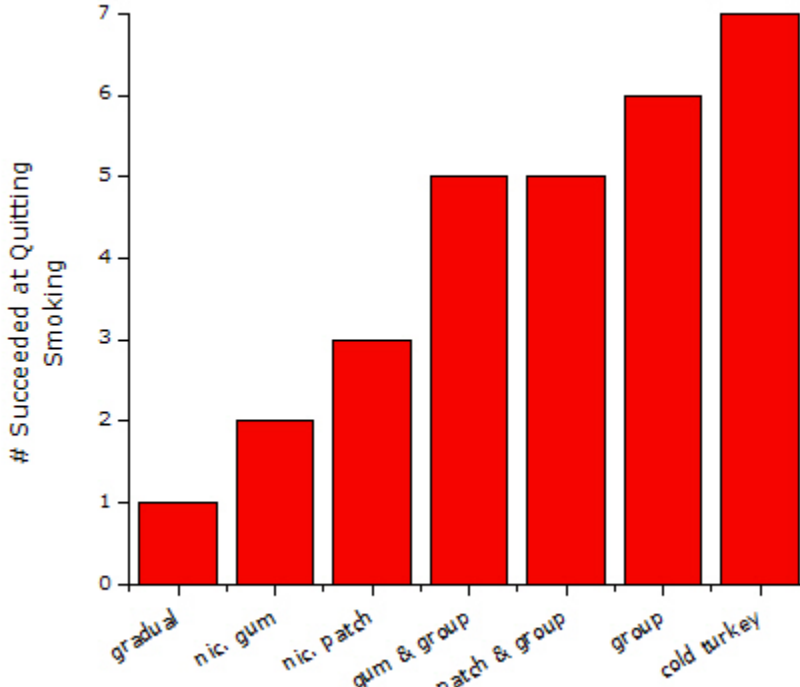
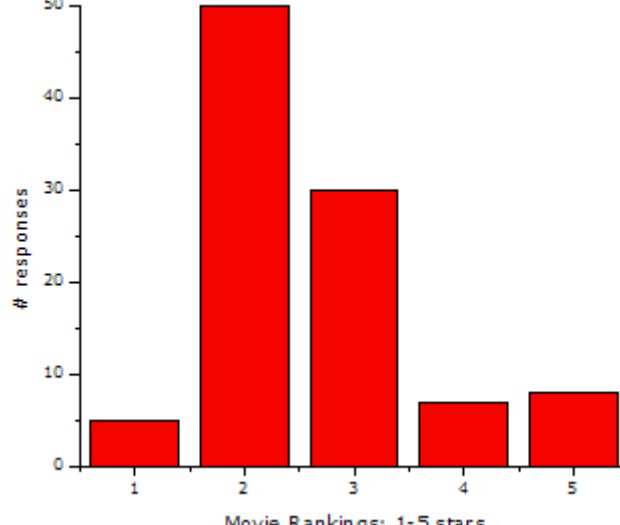


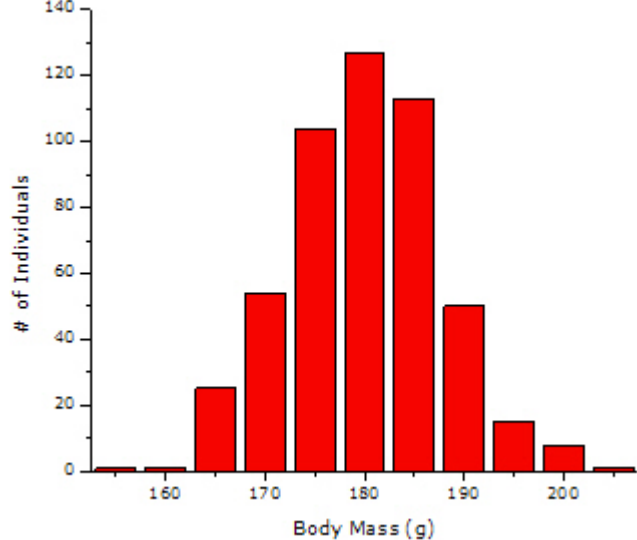
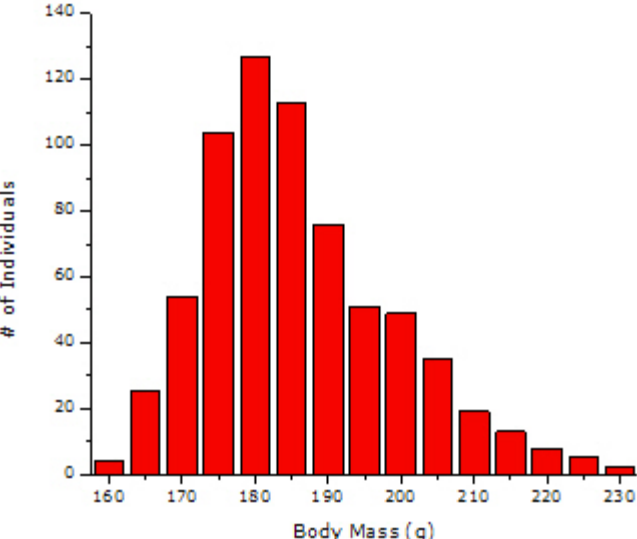
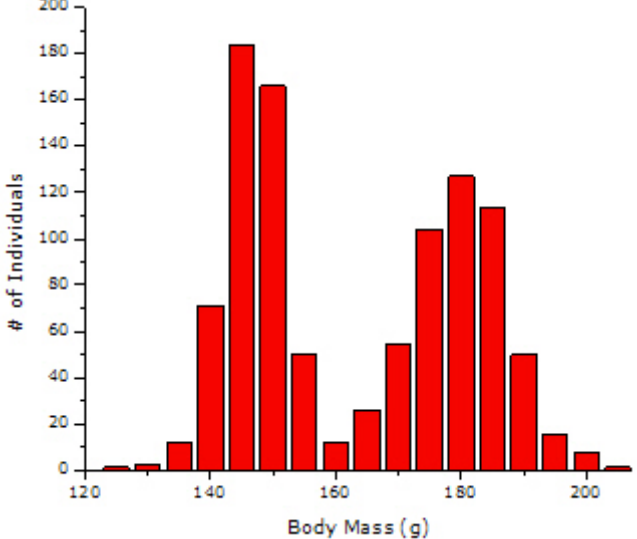
In general, the mean is the descriptive statistic most often used to describe the central tendency of a group of measurements. Of the three measures, it is the most sensitive measurement, because its value always reflects the contributions of each of the data values in the group. The median and the mode are less sensitive to "outliers"—data values at the extremes of a group. Imagine that, for the "without compost" group, the plant with the greatest number of leaves had 11 leaves, not 8. Both the median and the mode would remain unchanged. (Check for yourself and confirm that this is true.) The mean, however, would now be 5.5 instead of 5.0.

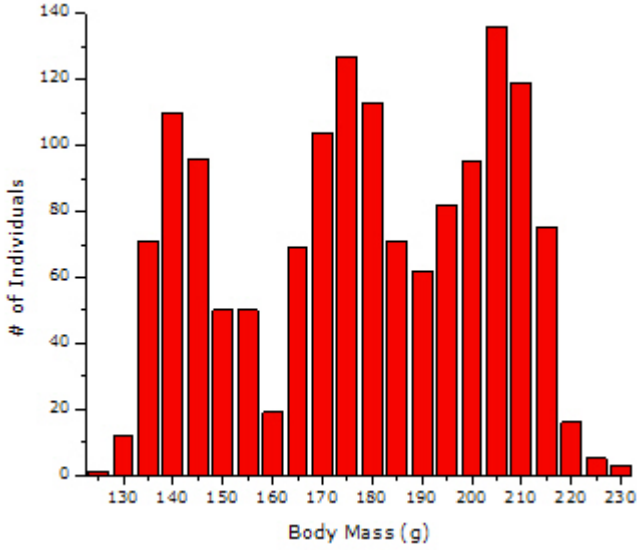
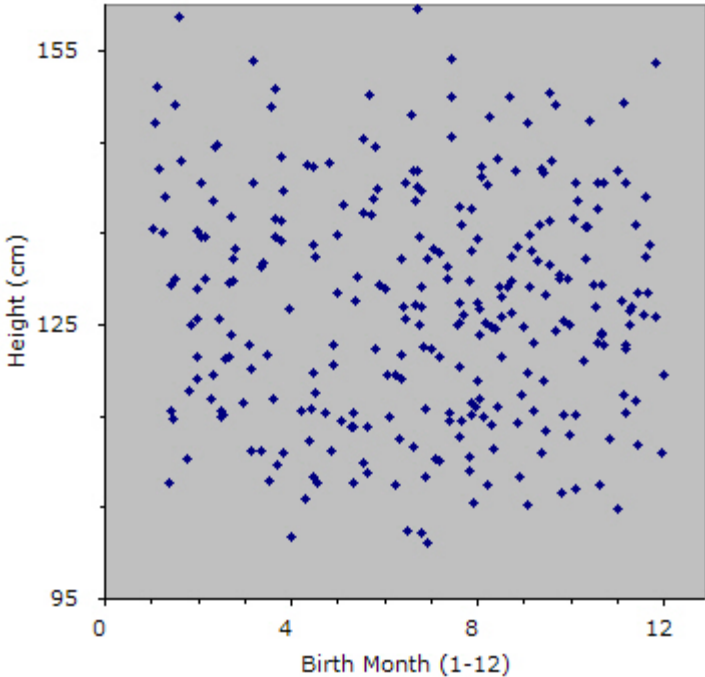
On the other hand, sometimes it is an advantage to have a measure of central tendency that is less sensitive to changes in the extremes of the data. For example, if your data set contains a small number of outliers at one extreme, the median may be a better measure of the central tendency of the data than the mean.

If your results involve categories instead of continuous numbers, then the best measure of central tendency will probably be the most frequent outcome (the mode). For example, imagine that you conducted a survey on the most effective way to quit smoking. A reasonable measure of the central tendency of your results would be the method that works most frequently, as determined from your survey.

It is important to think about what you are trying to accomplish with descriptive statistics, not just use them blindly. If your data contains more than one mode, then summarizing them with a simple measure of central tendency such as the mean or median will obscure this fact. Table 1, below, is a quick guide to help you decide which measure of central tendency to use with your data.

First, what are you trying to describe?	Second, what does your data look like?	Then, the best measure of central tendency is...																
Groups, or classes of things. Survey results often fall in this category, such as, "What is the most effective way to quit smoking?" or "Gender Differences in After-School Activities"	 <table><caption># Succeeded at Quitting Smoking</caption><thead><tr><th>Method</th><th># Succeeded</th></tr></thead><tbody><tr><td>gradual</td><td>1</td></tr><tr><td>nic. gum</td><td>2</td></tr><tr><td>nic. patch</td><td>3</td></tr><tr><td>gum & group</td><td>5</td></tr><tr><td>patch & group</td><td>5</td></tr><tr><td>group</td><td>6</td></tr><tr><td>cold turkey</td><td>7</td></tr></tbody></table>	Method	# Succeeded	gradual	1	nic. gum	2	nic. patch	3	gum & group	5	patch & group	5	group	6	cold turkey	7	Mode. In these made-up survey results, 'cold turkey' is the most frequent response.
Method	# Succeeded																	
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nic. gum	2																	
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Position on a ranking scale, such as: 1-5 stars for movies, books, or restaurants	 <table><caption># responses</caption><thead><tr><th>Movie Ranking</th><th># responses</th></tr></thead><tbody><tr><td>1</td><td>5</td></tr><tr><td>2</td><td>50</td></tr><tr><td>3</td><td>30</td></tr><tr><td>4</td><td>7</td></tr><tr><td>5</td><td>8</td></tr></tbody></table>	Movie Ranking	# responses	1	5	2	50	3	30	4	7	5	8	Median. The median movie ranking in this survey was 2.3 stars.				
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First, what are you trying to describe?	Second, what does your data look like?	Then, the best measure of central tendency is...
<p>Measures on a linear scale (e.g., voltage, mass, height, money, etc.)</p>	 <p>A histogram showing the distribution of body mass in grams. The x-axis is labeled 'Body Mass (g)' and ranges from 150 to 210 with major ticks every 10 units. The y-axis is labeled '# of Individuals' and ranges from 0 to 140 with major ticks every 20 units. The distribution is symmetrical and bell-shaped, centered around 180 grams. The bars are red.</p>	<p>Mean. The shape of this data is approximately the same on the left and the right side of the graph, so we call this <i>symmetrical data</i>. For symmetrical data, the mean is the best measurement of central tendency. In this case the mean body mass is 178 grams.</p>
<p>Measures on a linear scale (e.g., voltage, mass, height, money, etc.)</p>	 <p>A histogram showing the distribution of body mass in grams. The x-axis is labeled 'Body Mass (g)' and ranges from 150 to 230 with major ticks every 10 units. The y-axis is labeled '# of Individuals' and ranges from 0 to 140 with major ticks every 20 units. The distribution is right-skewed, with a peak around 185 grams and a long tail extending to the right. The bars are red.</p>	<p>Median. Notice how the data in this graph is non-symmetrical. The peak of the data is not centered, and the body mass values fall off more sharply on the left of the peak than on the right. When the peak is shifted like this to one side or the other, we call it <i>skewed data</i>. For skewed data, the median is the best choice to measure central tendency. The median body mass for this skewed population is 185 grams.</p>
<p>Measures on a linear scale (e.g., voltage, mass, height, money, etc.)</p>	 <p>A histogram showing the distribution of body mass in grams. The x-axis is labeled 'Body Mass (g)' and ranges from 120 to 200 with major ticks every 20 units. The y-axis is labeled '# of Individuals' and ranges from 0 to 200 with major ticks every 20 units. The distribution is bimodal, with two distinct peaks: one around 145 grams and another around 178 grams. The bars are red.</p>	<p>Notice how this graph has two peaks. We call data with two prominent peaks <i>bimodal data</i>. In the case of a bimodal distribution, you may have two populations, each with its own separate central tendency. Here one group has a mean body mass of 147 grams and the other has a mean body mass of 178 grams.</p>

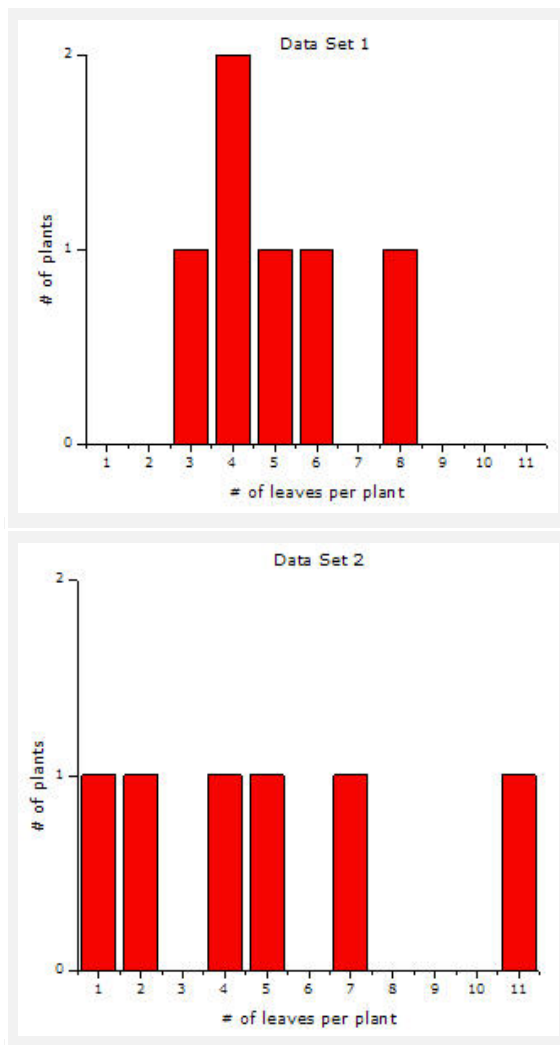
First, what are you trying to describe?	Second, what does your data look like?	Then, the best measure of central tendency is...
Measures on a linear scale (e.g., voltage, mass, height, money, etc.)	 <p>A histogram showing the distribution of body mass in grams. The x-axis is labeled 'Body Mass (g)' and ranges from 130 to 230 in increments of 10. The y-axis is labeled '# of Individuals' and ranges from 0 to 140 in increments of 20. The distribution is multimodal, with three prominent peaks: one at approximately 140g (height ~110), another at approximately 180g (height ~125), and a third at approximately 210g (height ~135). There is significant overlap between the tails of these peaks.</p>	None. Notice how this graph has three peaks and lots of overlap between the tails of the peaks. We call this <i>multimodal data</i> . There is no single central tendency. It is easiest to describe data like this by referring to the graph. Don't use a measure of central tendency in this case, it would be misleading.
Measures on a linear scale (e.g., voltage, mass, height, money, etc.)	<p>Boys' Height vs. Birth Month (simulated data)</p>  <p>A scatter plot titled 'Boys' Height vs. Birth Month (simulated data)'. The x-axis is labeled 'Birth Month (1-12)' and ranges from 0 to 12. The y-axis is labeled 'Height (cm)' and ranges from 95 to 155. The plot shows a dense collection of blue diamond-shaped data points scattered randomly across the entire area, with no discernible pattern or trend.</p>	None. In this case, the data is scattered all over the place. In some cases, this may indicate that you need to collect more data. In this case there is no central tendency.

Measures of Dispersion: Range, Variance, and Standard Deviation

Measures of central tendency describe the "average" of a data set. Another important quality to measure is the "spread" of a data set. For example, these two data sets both have the same mean (5):

data set 1: 3, 4, 4, 5, 6, 8
data set 2: 1, 2, 4, 5, 7, 11 .

Although both data sets have the same mean, it is obvious that the values in data set 2 are much more *scattered* than the values in data set 1 (see the graphs, below). For which data set would you feel more comfortable using the average description of "5"? It would be nice to have another measure to describe the "spread" of a data set. Such a measure could let us know at a glance whether the values in a data set are generally close to or far from the mean.



The descriptive statistics that measure the quality of *scatter* are called *measures of dispersion*. When added to the measures of central tendency discussed previously, measures of dispersion give a more complete picture of the data set. We will discuss three such measurements: the *range*, the *variance*, and the *standard deviation*.

Range

The *range* of a data set is the simplest of the three measures. The range is defined by the smallest and largest data values in the set. The range of data set 1 is 3–8. What is the range of data set 2?

The range gives only minimal information about the spread of the data, by defining the two extremes. It says nothing about *how* the data are distributed between those two endpoints. Two other related measures of dispersion, the variance and the standard deviation, provide a numerical summary of *how much* the data are scattered.

For more advanced material, see the "Variance & Standard Deviation" link on the www.sciencebuddies.org Project Guide's "Data & Analysis" page under "Related Links."